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Calculations on the radio emission resulting from geomagnetic charge separation in an extensive air shower

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Abstract. Calculations have been made of the radio emission resulting from the separation of extensive air shower electrons and positrons in the earth's magnetic field. Frequency spectra and lateral distributions of the radiation are presented for showers developing at different heights in the atmosphere.

1. Introduction

It has been demonstrated both experimentally (Prescott *et al* 1971, Hough and Prescott 1970a, Vernov *et al* 1970, Allan *et al* 1970a) and theoretically (Colgate 1967, Fujii and Nishimura 1970, Charman and Jelley 1968, Allan 1970, Castagnoli *et al* 1969) that the geomagnetic charge separation of electrons and positrons in an extensive air shower is the dominant process in radio pulse production. The calculations reported here are solely concerned with this particular emission mechanism.

Two basic approaches have been adopted in calculating the radio emission from air showers. Kahn and Lerche (1966), Fujii and Nishimura (1970, 1971), Castagnoli *et al* (1969) and Spencer (1970) have solved Maxwell's equation for varying shower models; the degree of sophistication of the electron distributions used being necessarily limited by the mathematical complexity involved. Allan (1971a), employing an entirely different approach, has used the formulation of Feynman (1963) for the radiation from a relativistically moving charged particle:

$$E = -\frac{e}{c^2}\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2},$$

where E is the field strength, e is the electronic charge, c is the velocity of light and $\mathrm{d}^2\theta/\mathrm{d}t^2$ is the angular acceleration of the particle as viewed by an observer at the retarded time t.

2. Electronic shower development

The work reported here forms part of a much wider study of extensive air shower models which is being carried out at Durham University. The model used for the calculations reported here is quite conventional (see for example the work of de Beer *et al* 1966,

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Hillas 1971). The nucleon and pion interaction mean free paths were taken as 80 g cm^{-2} and 120 g cm^{-2} respectively with corresponding inelasticity values of 0·50 and 1·0; the multiplicity of produced secondaries increased as $E^{1/4}$ where E is the energy of interaction.

The successive interactions of a primary cosmic ray proton were followed using Monte Carlo techniques. In particular, the points of interaction in the atmosphere were chosen randomly giving showers with varying position of shower maximum. The meson cascade was followed step by step through the atmosphere (Hillas 1965), and provided an energy by height matrix of neutral pions whose decay photons initiate the electromagnetic cascade. The development of this cascade was followed through the atmosphere using one-dimensional approximation A theory (Rossi 1965). Electrons and photons whose energy fell below 50 GeV were removed from the cascade and were subsequently used to generate three-dimensional electron showers. Details of the latter were obtained from the tabulated results of electron shower development given by Messel and Crawford (1970) using Monte Carlo simulations of electron and photon initiated cascades. These workers produced secondary electron energy, lateral and longitudinal distributions for electron and photon primary energies below 50 GeV. The secondary electron distributions were folded with the electron and photon primary spectra produced via approximation A.

The resulting electron distribution was stored as a matrix with dimensions of atmospheric height, radial distance and energy. Forty 25·8 g cm^{-2} height intervals, twenty-five $\frac{1}{4}$-decade radial intervals and ten $\frac{1}{4}$-decade energy intervals (these included electrons with energies between approximately 1·3 MeV and 450 MeV) were used.

From many showers, each generated by a vertically incident cosmic ray, eight electron cascades were selected which differed only in the position of shower maximum, varying between 500 g cm^{-2} and 900 g cm^{-2} atmospheric depth. The total number of electrons at shower maximum was $6{\cdot}0 \times 10^7$, corresponding to a primary cosmic ray energy 10^{17} eV. Four of the electron cascades used are shown in figure 1.

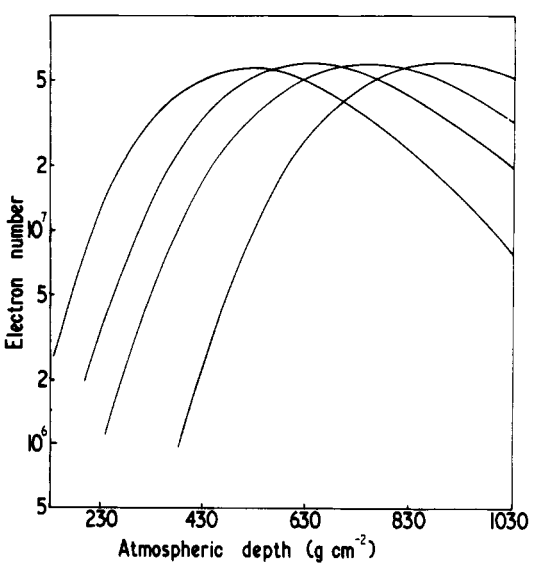

Figure 1. The variation of shower electron number with atmospheric depth. Four representative cascades are shown.

3. Method of calculating the radio emission

The time measured with respect to the arrival of the shower front at sea level, at which an observer views the geomagnetic deflections of the electrons is the sum of the geometrical time delays due to obliquity and the electron's lag behind the shower front resulting from Coulomb scattering and the geomagnetic deflection itself. Time delays were corrected for the effect of the atmospheric refractive index by assuming that the refractive index at a height h km in the atmosphere is given by (Jelley 1958)

$$\eta(h) = 1 + \eta_0 \exp - \left(\frac{h}{\lambda}\right)$$

with $\lambda = 7{\cdot}1$ km and $\eta_0 = 2{\cdot}1 \times 10^{-4}$. Time delays resulting from the Coulomb scattering of electrons of known energy and radial displacement were kindly supplied by Dr D J Marsden of Leeds University. These had been calculated using a Monte Carlo simulation of an electron–photon cascade. Only electron time delays at sea level were, however, available and it was therefore assumed that the electron lags in the atmosphere were inversely proportional to atmospheric pressure (Cocconi 1954). Some of the representative time delays are shown in the work of Allan (1971b).

The additional lateral displacements D_g and time delays L_g due to the geomagnetic deflections of the electrons were estimated as follows:

$$D_g = \frac{0{\cdot}32D}{P} \quad \text{and} \quad L_g = \frac{0{\cdot}05L}{P}$$

where D and L are the known lateral displacements and time delays of the electron and P is the atmospheric pressure in atmospheres (Allan 1971a).

In order to reduce the computation involved all electrons were considered to lie in the east–west plane with a horizontal north–south magnetic field of strength 0·3 G. This restriction on the shower geometry will lead to a slight steepening of the calculated lateral distribution given later in the paper.

The geomagnetic angular deflections of all shower electrons as viewed by observers at distances along the east–west axis of 5 m to 300 m from the shower axis were calculated and binned into 1 ns intervals for a period of 200 ns. The frequency spectrum of the emitted radiation is then given by the Fourier transform of the second derivative of the summed angular deflections.

The lateral distribution of the radiation was calculated from the frequency spectra found at the different radial distances.

The electron to positron number ratio as a function of particle energy was based on the calculations of Fujii and Nishimura (1970).

4. Frequency spectrum

Figure 2 shows the frequency spectra for a shower with its maximum development at 700 cm^{-2}; spectra for radial distances 5 m to 300 m are included.

The field has a maximum between 40 MHz and 80 MHz for radial distances up to about 150 m. Beyond this distance the maximum occurs at progressively lower frequencies. The present results can be compared with the experimental data of Allan *et al* (1971a, b). These workers observed electric fields at 100 m of 2·7, 6·5 and 2·7 μV

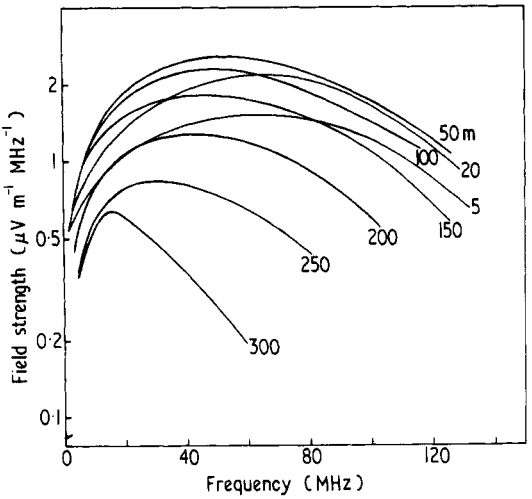

Figure 2. Frequency spectra of the radiation for a shower with its maximum development at 700 g cm^{-2}; spectra for radial distances 5 m to 300 m are included.

m^{-1} MHz^{-1}, at frequencies of 32, 60 and 105 MHz respectively for showers of primary energy 10^{17} eV. Although the corresponding calculated values of 1·6, 2·3 and 1·5 μV m^{-1} MHz^{-1} are somewhat lower, there is general agreement with the spectral shape. The continued falling of the frequency spectrum below 15 MHz is, however, in sharp disagreement with observations made between 2 and 4 MHz where fields up to 100 times the measured value in the 30 to 100 MHz band have been reported†. As noted recently by Allan (1972) it is difficult to reconcile the generally accepted view of electron shower development with the observation of such large fields at low frequencies. Clay (1972) calculates a steadily increasing field at low frequencies, in contrast to the present work, which he attributes to the large number of low energy electrons ($E \gtrsim 50$ MeV) present in the shower. Although the median energy of electrons in the shower is indeed low (~ 15 MeV), we would argue that their overall contribution is not large because in calculating the shower dipole moment account must be taken of the relatively large delays associated with such electrons.

Showers having their maximum development above 700 g cm^{-2} have similar frequency spectra but with the maximum occurring at somewhat higher frequencies for a fixed radial distance. Conversely, low developing showers have their maximum field at a lower frequency. These differences in the frequency spectra reflect the way in which a shower developing high in the atmosphere appears, to an observer at ground level, to be compressed into a smaller time interval than a low developing shower, that is, the rise time of the shower electric dipole moment is shorter.

5. Lateral distributions

Figures 3, 4 and 5 show the lateral distribution of the radiation at frequencies of 30, 60 and 100 MHz respectively. The spectra for showers having their maximum development at 500, 600, 700, 800 and 900 g cm^{-2} are included. Beyond 100 m, where the lateral

† Allen *et al* (1970b), Prescott *et al* (1970), Stubbs (1971), Hough *et al* (1971).

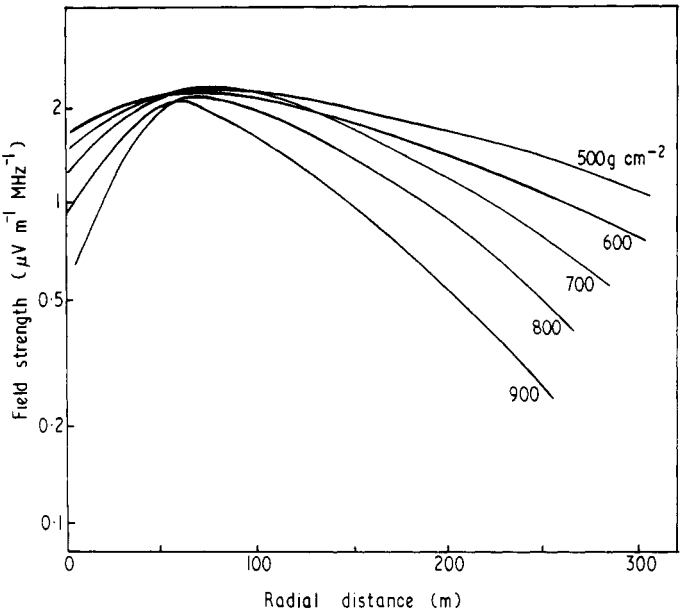

Figure 3. Lateral distributions of the radiation at a frequency of 30 MHz. The spectra for showers having their maximum development at 500, 600, 700, 800 and 900 g cm^{-2} are shown.

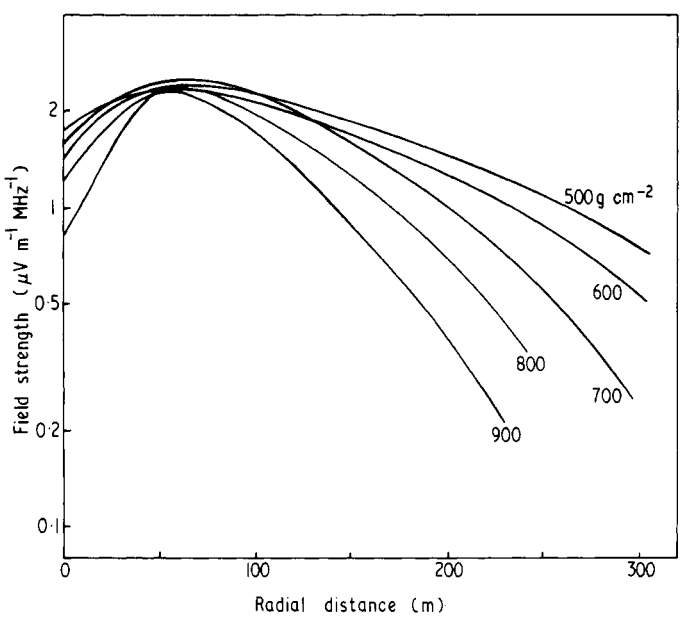

Figure 4. Lateral distributions of the radiation at a frequency of 60 MHz. The spectra for showers having their maximum development at 500, 600, 700, 800 and 900 g cm^{-2} are shown.

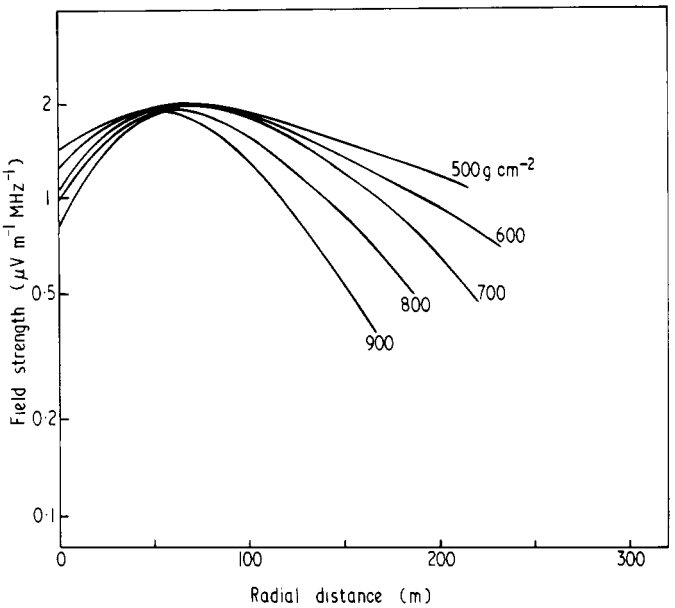

Figure 5. Lateral distributions of the radiation at a frequency of 100 MHz. The spectra for showers having their maximum development at 500, 600, 700, 800 and 900 g cm^{-2} are shown.

distribution of the radiation strongly reflects the geometrical configuration of the electron longitudinal distribution as seen by the observer, the following effects can be seen. Showers developing lower in the atmosphere have steeper lateral distributions than higher developing showers. Also, for a given shower the lateral distribution becomes steeper as the frequency increases since the part of the shower which contributes coherently to the radiation becomes progressively smaller.

On axis the observed field depends not only on the longitudinal electron distribution but also on the lateral distribution of the electrons (see later comments), their time delay due to scattering and the refractive index of the atmosphere. As the radial distance is increased the additional delays due to obliquity become important, and, as noted by Allan (1971c) there is a minimum in the time delays as observed at a fixed radial distance for sources at varying heights in the atmosphere. This has the consequences that at radial distances of about 50 m the observed field is moderately insensitive to the position of shower maximum.

The height of maximum development of the electron cascade is of primary importance in understanding the nature of the nuclear cascade initiated by the primary cosmic ray. It is, therefore, instructive to demonstrate the lateral distributions in a different way. Figures 6 and 7 show the field at different radial distances as a function of depth of shower maximum for frequencies of 30 and 60 MHz respectively.

The field at 50 m, which has been shown to be insensitive to the position of shower maximum, is clearly an important parameter in determining the total energy of the shower. As the radial distance increases then the field becomes progressively more sensitive to the position of shower maximum. Unfortunately, the field strength itself decreases and the background radio noise gives a natural limit to the precision with which the longitudinal development of the shower can be investigated using radio techniques.

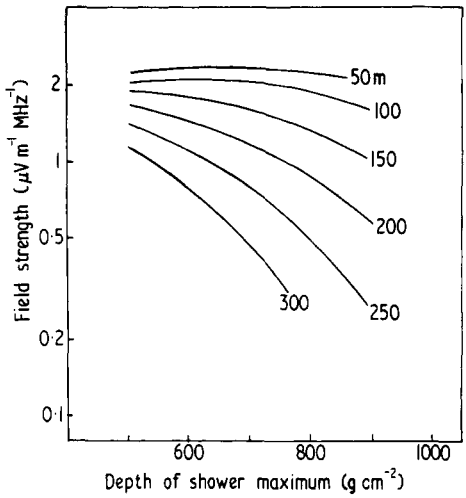

Figure 6. The variation of the electric field at a frequency of 30 MHz with depth of shower maximum; radial distances from 50 m to 300 m are shown.

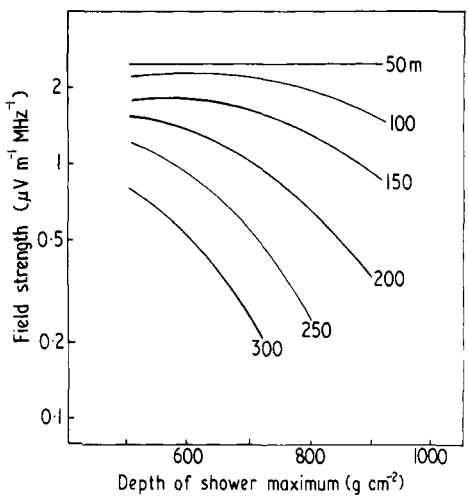

Figure 7. The variation of the electric field at a frequency of 60 MHz with depth of shower maximum; radial distances from 50 m to 300 m are shown.

Checks were made on the sensitivity of the radio emission to changes in the lateral distributions of the electrons. Figure 8 shows the resulting distributions for a shower with maximum electron development at 700 g cm^{-2} depth and at a radio frequency of 60 MHz. Curve A corresponds to electrons with a lateral displacement artificially increased by a factor of two, curve B to the normal electron displacement and curve C to zero displacement. As indicated earlier the shape of the radio lateral distribution within 100 m of the shower axis is sensitive to the lateral distribution of the electrons. Beyond 100 m, however, the changes are not as marked with the lateral distributions flattening slightly for increasing electron radial displacement.

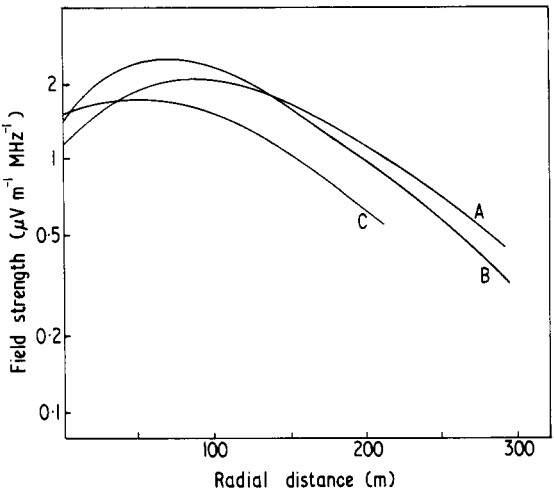

Figure 8. The variation of the lateral distribution of the emitted radiation with artificial increases and decreases in the scale of the shower electron lateral displacements. Curve A, electron lateral displacement increased by a factor of 2; curve B, normal displacement; curve C, zero displacement. Frequency of shower 60 MHz; maximum depth of shower 700 g cm^{-2}.

Experimental results on the lateral distribution of the radio emission do not, as yet, appear to give a clear picture, particularly for small radial distances. Khristiansen *et al* (1971) at a frequency of 30 MHz and for showers with a total number of electrons greater than or equal to 10^7 find that the fluctuations are large near the axis and that for showers containing a high proportion of muons the field first increased with increasing radial distance before decreasing at about 100 m. The Haverah Park group (Allan *et al* 1971a, b) for showers of energy greater than about 10^{17} eV and at frequencies of 32, 60 and 105 MHz find that the lateral distribution for radial distances 50 m to 200 m is approximately exponential and that the steepness of the distribution increases with increasing frequency. The Calgary Group (Hough and Prescott 1970a, b, Clay *et al* 1971) find for showers of energy greater than 2×10^{15} eV and at 22 MHz that the lateral distribution falls steeply as $R^{-0\cdot5}$ for radial distances R up to about 50 m and then falls less steeply for radial distances up to 300 m.

The calculated results on the lateral distributions of the radiation would only appear to be in general agreement with the experimental results for radial distances greater than 50 m. It should be noted, however, that there are few close-core measurements and the experimental data in this region are not as yet complete. There is good agreement between the predicted results and the experimental data on the frequency dependence of the lateral distribution of the radiation.

6. Conclusions

It has been demonstrated that information on the nature of extensive air showers should be obtainable from the study of their radio emission. The electric field at about 50 m from the shower axis is relatively insensitive to the height of maximum shower development and can thus be used as a useful indicator of the shower energy. Beyond 100 m

the electric field becomes progressively more sensitive to the height of shower maximum and thus tends to confirm the earlier hopes that the radio technique can be used to provide information on the longitudinal development of the shower.

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